FLUID MECHANICS

Tank Drainage

Watch

Practice

Master

Instructions

How to Use This Workbook

This workbook covers transient tank drainage and transient tank equalization problems.

The problems are carefully structured and progressively increase in difficulty. It is therefore recommended to work through them in the order presented.

All constants, material properties, and equations required to solve the problems are provided in the reference section.

How to Use the QR Codes

Detailed solution videos are published on YouTube over time.

- **Printed version:** Scan the QR code to open the corresponding video.
- **Digital version:** Click the QR code to open the corresponding video.

The videos can be used both while working through the problems and for review afterward.

Watch Detailed Solutions on YouTube

Engineering | With Erwin

Contents

Symbols

Roman Symbols

Symbol	Unit	Description
A	$\mathrm{m^2}$	Area, cross-sectional area
b	m	Width
C	–	Coefficient
d, D	m	Diameter
g	$\mathrm{m\,s^{-2}}$	Gravitational acceleration
h, H	m	Height, elevation difference
m	kg	Mass
$\dot{m}$	$\mathrm{kg\,s^{-1}}$	Mass flow rate
p	Pa	Pressure
Q	$\mathrm{m^3\,s^{-1}}$	Volumetric flow rate
r, R	m	Radius
t	s	Time
v	$\mathrm{m\,s^{-1}}$	Velocity
V	$\mathrm{m^3}$	Volume
x, y, z	m	Spatial coordinates

Greek Symbols

Symbol	Unit	Description
Δ	–	Difference, change
π	–	Pi
ρ	$\mathrm{kg\,m^{-3}}$	Density

Subscripts

Symbol	Description
∞	Ambient conditions
1, 2, 3, ...	State identifier
d	Discharge
in	Inlet
out	Outlet
ideal	Ideal, lossless
actual	Actual, including losses
w	Water

Formula Sheet

Constants

Gravitational acceleration	$g = 9.81$	$\mathrm{m\,s^{-2}}$
Pi	$\pi = 3.1416$	–

Fluid Properties

Density of water	$\rho_{\mathrm{w}} = 10^3$	$\mathrm{kg\,m^{-3}}$

Note: Unless stated otherwise, the values listed above shall be used throughout this workbook. They represent simplified engineering approximations suitable for the exercises presented here.

Fundamental Equations

Bernoulli Equation (pressure form)	$p + \rho \cdot \frac{v^2}{2} + \rho \cdot g \cdot h = \text{constant}$
Discharge Coefficient	$C_d = \frac{v_{\text{actual}}}{v_{\text{ideal}}}$
Conservation of Mass	$\frac{dm}{dt} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}}$
Mass	$m = \rho \cdot V$
Mass Flow Rate	$\dot{m} = \rho \cdot Q$
Volumetric Flow Rate	$Q = A \cdot v$
Diameter	$d = 2 \cdot r$
Circle Area	$A_{\text{circle}} = \frac{\pi \cdot d^2}{4}$
Cone Volume	$V_{\text{cone}} = A_{\text{circle}} \cdot \frac{h}{3}$
Cylinder Volume	$V_{\text{cylinder}} = A_{\text{circle}} \cdot b$
Sphere Volume	$V_{\text{sphere}} = \frac{\pi \cdot d^3}{6}$

Trigonometric Relationships

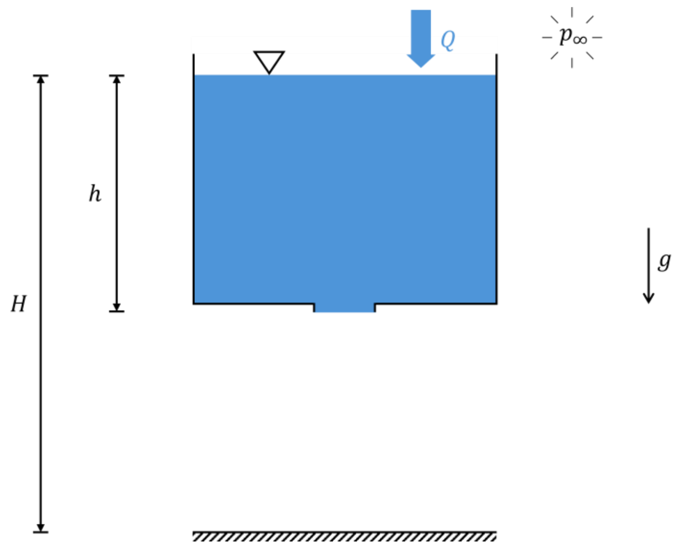
$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$	$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$
$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$	$a^2 = b^2 + c^2$

Exercises

Exercise 1 – Steady Tank Discharge

A large, open water tank is fitted with a circular outlet at its bottom with a diameter of $d_{\text{out}} = 10$ cm. A continuous inflow maintains a constant water level inside the tank. Initially, the vertical distance between the free surface and the outlet is $h = 4$ m, while the distance between the free surface and the ground is $H = 9$ m.

Assume that the flow is initially ideal and frictionless.



a) Using the Bernoulli equation, determine the ideal exit velocity of the water jet.

b) Taking the discharge coefficient into account, determine the actual exit velocity for $C_d = 0.80$.

c) Determine the volumetric flow rate Q required to maintain a constant water level in the tank.

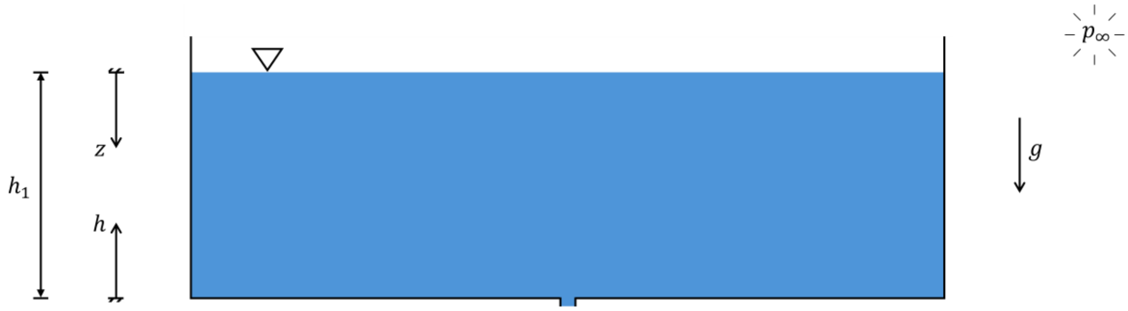
d) Determine the impact velocity of the water jet when it reaches the ground. Air resistance may be neglected.

e) Sketch the water jet in the figure in a qualitatively and physically consistent manner.

Exercise 2 – Draining a Tank

A large, open water tank has a constant cross-sectional area of $A = 9\ \mathrm{m}^2$. An outlet with an area of $A_{\mathrm{out}} = 0.2\ \mathrm{m}^2$ is located at the bottom of the tank. Initially, the outlet is closed. The vertical distance between the free surface and the outlet is $h_1 = 5\ \mathrm{m}$. After the outlet is opened, the water level decreases under unsteady conditions.

Given: $C_d = 0.8$



a) Determine the exit velocity, $v(h)$ as a function of the water height h.

b) Using conservation of mass, derive the differential equation describing the evolution of the water level, $h(t)$ and rearrange it into the form $dt = f(h)dh$.

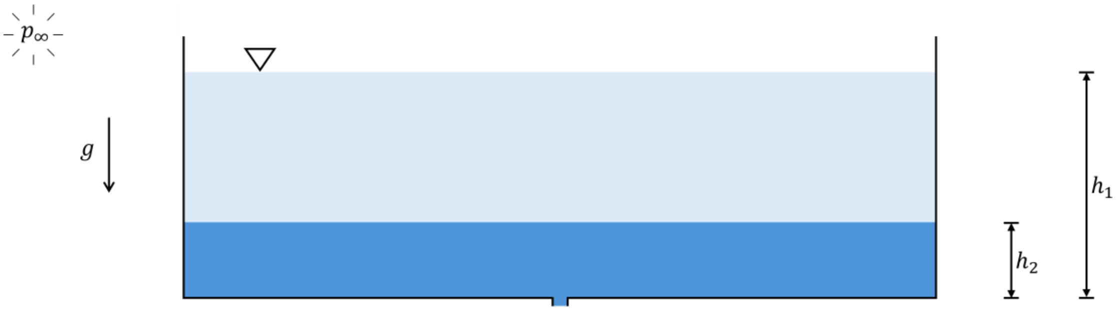
c) Determine the time required for the tank to empty completely.

d) Sketch the qualitative variation of the exit velocity, $v(t)$, with time.

Exercise 3 – Draining to a Specified Water Level

A large, open water tank has a constant cross-sectional area of $A = 6\ \mathrm{m}^2$. An outlet with an area of $A_{\mathrm{out}} = 0.12\ \mathrm{m}^2$ is located at the bottom of the tank. Initially, the vertical distance between the free surface and the outlet is $h_1 = 8$ m. After the outlet is opened, the water level decreases under unsteady conditions.

Given: $C_d = 0.82$



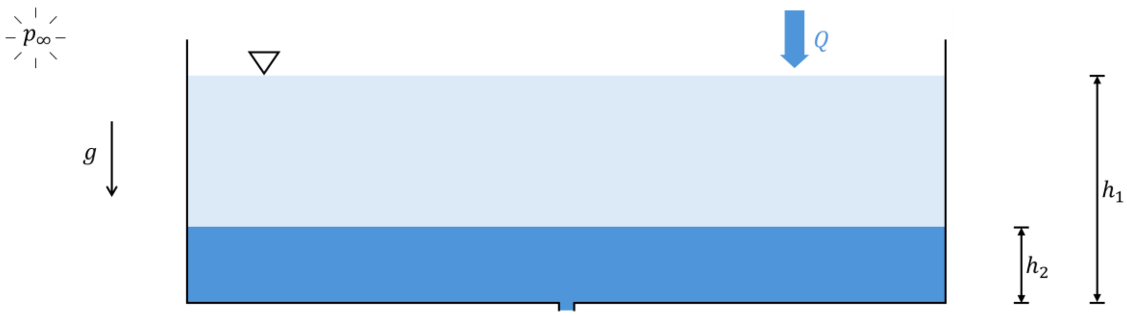
a) Determine the time required for the water level to decrease to $h_2 = 2$ m.

b) Determine the water level after 20 seconds.

Exercise 4 – Draining a Tank with Inflow

A large, open water tank has a constant cross-sectional area of $A = 12\ \mathrm{m}^2$. An outlet with an area of $A_{\mathrm{out}} = 0.2\ \mathrm{m}^2$ is located at the bottom of the tank. Initially, the vertical distance between the free surface and the outlet is $h_1 = 12\ \mathrm{m}$. After the outlet is opened, the water level decreases under unsteady conditions while water enters the tank at a constant volumetric flow rate of $Q = 1.5\ \mathrm{m}^3/\mathrm{s}$.

Given: $C_d = 0.90$



a) Determine the steady-state water level.

b) Determine the time required for the water level to decrease to $h_2 = 6\ \mathrm{m}$.

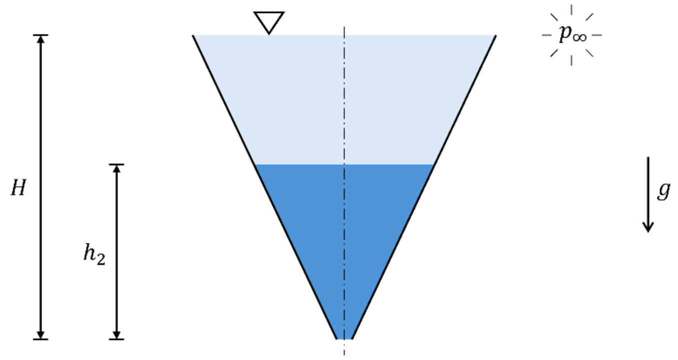
Hint:

$$\int \frac{u}{au + b}\,du = \frac{u}{a} - \frac{b}{a^2}\ln|au + b|$$

Exercise 5 – Draining a Conical Tank

A large, open water tank has the shape of a right circular cone. A circular outlet with a diameter of $d_{\text{out}} = 5$ cm is located at the lowest point of the tank. Initially, the tank is completely filled with water $h_1 = H = 2$ m. The upper diameter of the tank is $D = 1$ m. After the outlet is opened, the water level decreases under unsteady conditions. The diameter of the free surface is given by $d(t) = D \cdot \frac{h(t)}{H}$.

Given: $C_d = 0.84$



a) Determine the time required for the tank to empty completely.

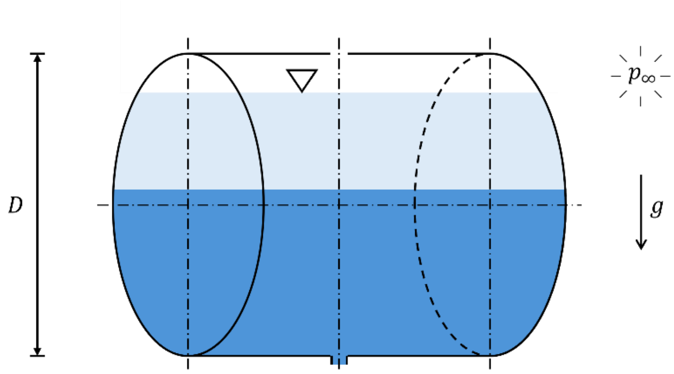
b) Determine the exit velocity of the water after 30 seconds.

c) The tank geometry was simplified as part of the modeling process. Explain this simplification and discuss its influence on the result obtained in part a).

Exercise 6 – Draining a Horizontal Cylindrical Tank

A large, open water tank has the shape of a horizontal cylinder with a circular cross-section. A circular outlet with a diameter of $d_{\text{out}} = 12$ cm is located at the lowest point of the tank. The tank diameter is $D = 4$ m. The tank width is $b = 6$ m. Initially, the water level is $h_1 = 3.25$ m. After the outlet is opened, the water level decreases under unsteady conditions.

Given: $C_d = 0.80$



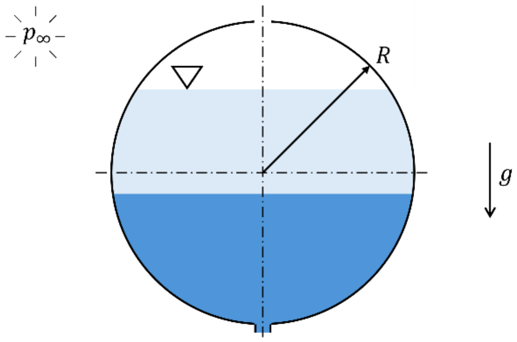
a) Derive an expression for the cross-sectional area, $A(h)$, as a function of the water level h.

b) Determine the time required for the water level to decrease to $h_2 = 1$ m.

Exercise 7 – Draining a Spherical Tank

A large, open spherical tank has a circular outlet with a diameter of $d_{\text{out}} = 14$ cm located at the lowest point of the tank. The tank radius is $R = 3$ m. Initially, the water level is $h_1 = \frac{3}{2}R$. After the outlet is opened, the water level decreases under unsteady conditions.

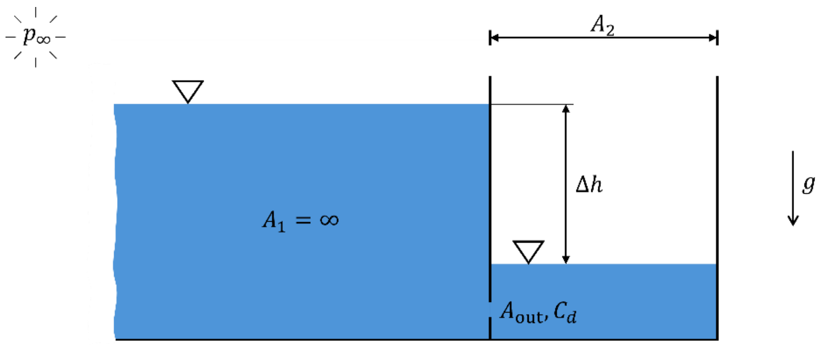
Given: $C_d = 0.86$



a) Determine the time required for the water level to decrease to $h_2 = \frac{1}{2}R$.

Exercise 8 – Level Equalization with a Large Reservoir

A very large upstream reservoir supplies water to a basin through a connecting opening. The cross-sectional area of the basin is A_2. The connecting opening has a cross-sectional area of A_{out}. Initially, the difference between the two water levels is H. After the connecting opening is opened, water flows from the upstream reservoir into the basin. An unsteady level equalization process takes place until both water levels become equal.



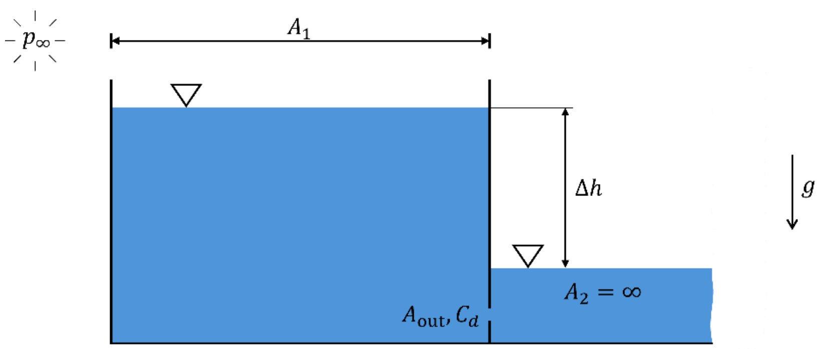
a) Derive a general expression $t = f(H)$ for the time required for complete level equalization.

b) Determine the time required for complete level equalization for:

$A_1 = \infty, A_2 = 1250\ \text{m}^2, A_{\text{out}} = 0.03\ \text{m}^2, H = 3.5\ \text{m}, C_d = 0.87$

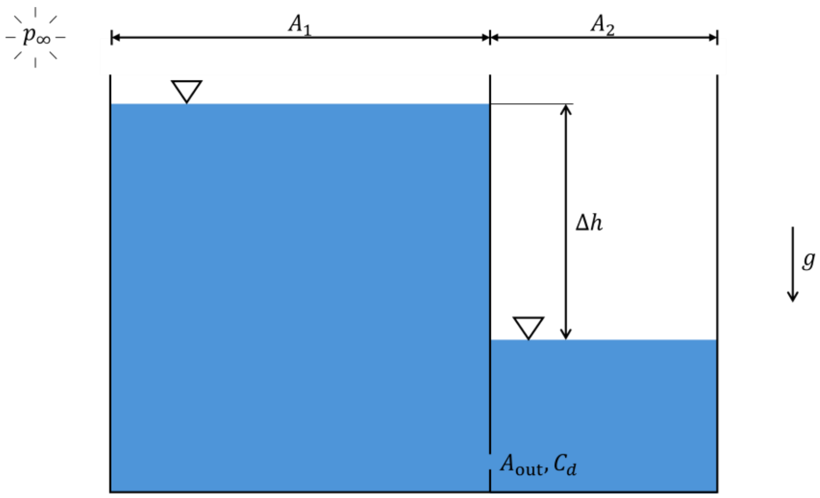
c) For the reverse case, in which the basin drains into a very large downstream reservoir, determine the level difference Δh after 3 hours.

$A_1 = 1250\ \text{m}^2, A_2 = \infty$



Exercise 9 – Level Equalization Between Communicating Tanks

Two large, open tanks are connected by a common opening. The cross-sectional areas of the tanks are A_1 and A_2. The connecting opening has a cross-sectional area of A_{out}. Initially, the level difference between the two free surfaces is H. After the connecting opening is opened, water flows from the left tank into the right tank. An unsteady level equalization process takes place until both free surfaces reach the same level.



a) Derive a general expression $t = f(H)$ for the time required for complete level equalization.

b) Determine the time required for complete level equalization for
$A_1 = 25\ \text{m}^2, A_2 = 10\ \text{m}^2, A_{\text{out}} = 0.01\ \text{m}^2, H = 6\ \text{m}, C_d = 0.8$

c) Determine the level difference Δh after 5 minutes.

Answer Key

Exercise 1 – Steady Tank Discharge

a) $v_{1,\text{ideal}} = 8.8589\ \frac{\text{m}}{\text{s}}$

b) $v_{1,\text{actual}} = 7.0871\ \frac{\text{m}}{\text{s}}$

c) $Q = 0.0557\ \frac{\text{m}^3}{\text{s}}$

d) $v_{2,\text{actual}} = 12.179\ \frac{\text{m}}{\text{s}}$

e) See detailed video solution

Exercise 2 – Draining a Tank

a) $v(h) = C_d \cdot \sqrt{2 \cdot g \cdot h}$

b) $dt = -\frac{A}{C_d \cdot A_{\text{out}} \cdot \sqrt{2 \cdot g}} \cdot \frac{1}{\sqrt{h}} dh$

c) $t(h = 0) = 56.7821\ \text{s}$

d) See detailed video solution

Exercise 3 – Draining to a Specified Water Level

a) $t(h = 2\ \text{m}) = 38.936\ \text{s}$

b) $h(t = 20\ \text{s}) = 4.4184\ \text{m}$

Exercise 4 – Draining a Tank with Inflow

a) $h = 3.5395\ \text{m}$

b) $t(h_2 = 6\ \text{m}) = 88.5606\ \text{s}$

Exercise 5 – Draining a Conical Tank

a) $t(h = 0) = 60.8144\ \text{s}$

b) $v(t = 30\ \text{s}) = 4.593\ \frac{\text{m}}{\text{s}}$

c) See detailed video solution

Exercise 6 – Draining a Horizontal Cylindrical Tank

a) $A(h) = 2 \cdot \sqrt{(2R - h) \cdot h} \cdot b$

b) $t(h_2 = 1\ \text{m}) = 907.5866\ \text{s}$

Exercise 7 – Draining a Spherical Tank

a) $t(h_2 = \frac{1}{2}R) = 790.481\ \mathrm{s}$

Exercise 8 – Level Equalization with a Large Reservoir

a) $t = \frac{A_2}{C_d \cdot A_{\mathrm{out}}} \cdot \sqrt{\frac{2 \cdot H}{g}}$

b) $t(\Delta h_2 = 0) = 40456.1001\ \mathrm{s}$

c) $\Delta h(t = 3\ \mathrm{h}) = 1.8807\ \mathrm{m}$

Exercise 9 – Level Equalization Between Communicating Tanks

a) $t = \frac{A_1 \cdot A_2}{C_d \cdot A_{\mathrm{out}} \cdot (A_1 + A_2)} \cdot \sqrt{\frac{2 \cdot H}{g}}$

b) $t(\Delta h_2 = 0) = 987.5023\ \mathrm{s}$

c) $\Delta h(t = 5\ \mathrm{min}) = 2.9082\ \mathrm{m}$

Watch Detailed Solutions on YouTube

Found a mistake or have an idea for a new problem?

Feel free to email us at: feedback@witherwin.com

Engineering | With Erwin